From Rational Expectations to Market Microstructure

By Albert S. Kyle

University of Maryland
Robert H. Smith School of Business

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Plan for 3 Lectures

- Lecture #1: Single-period rational expectations
- Lecture #2: Dynamic rational expectations: continuous models
- Lecture #3: Modeling value and growth (different topic)

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"Rational Expectations" versus "Behavioral Finance"

- **Rational Expectations == Correct use of Information**
  - Traders use correct models
  - Traders use correctly the information they have
    - But may not have all information
  - Share common prior
    - Use Bayes Law correctly
- **Behavioral Finance == Incorrect use of Information**
  - Trade based on "psychology," not "information" => "Behavioral Biases"
    - Does psychology generate systematic departures from rationality?
  - Traders are "irrational"
    - "overconfident," "optimistic"
  - Do not share common prior
    - But may use Bayes Law correctly, or incorrectly
    - If potentially irrational traders use Bayes Law correctly, then whether traders are rational or rational is an empirical issue

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"Market Efficiency" versus "Anomalies"

- **Market Efficiency**
  - Expected returns, conditional on available information, provide an appropriate reward for bearing risk
    - Prices appear to fluctuate randomly
  - It is hard to make money by beating the market
    - Hedge fund industry?
- **Anomalies == Market Inefficiency**
  - Reward for bearing risk, conditional on available information, seems too large to be consistent with models
    - But what models are we talking about?

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"Limits to Arbitrage"

- Rational expectations == Market efficiency?
  - Maybe
- Behavioral Finance == Market Inefficiency?
  - Definitely not (Friedman)
    - Even if irrational traders try to push prices away from efficient levels …
    - … Arbitragers should "arbitrage away" inefficiencies
- Behavioral Finance => Inefficiency only if there are limits to arbitrage
  - What limits arbitrage?
    - Capital, or wealth (Is $2 trillion in hedge funds enough?)
    - Transactions costs (including 2%/20% hedge fund fees)
    - Regulations and other barriers to trade

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"Market Microstructure"

- Term coined by Mark Garmon and/or Nils Hakannson
- Models how trading takes place:
  - Rules of the trading game
    - Floor trading "eye-ball to eye-ball"
    - Electronic trading platforms with algorithmic tradering:
  - Focus on short time periods, perhaps seconds, even micro-seconds.
  - Related to transactions cost measurement

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Rational Expectations and Market Efficiency

- Keynes
  - Beauty contest: markets not efficient
  - "Treatise on Money" : failed attempt at rational expectations
  - Traders rewarded for bearing risk
- Holbrook Working (1940's and 1950's): prices should fluctuate randomly if information used correctly
- Milton Friedman: Destabilizing speculation might be beneficial
- John Muth
  - Father of rational expectations
  - Interesting quadratic approach
- Euguen Fama: popularized efficient markets, a-theoretically
  - Big impact, but positive or negative?
- Steven Leroy: importance of risk aversion
- Robert E. Lucas, Jr.: explained importance of rational expectations with clean models and macro applications
- Grossman: Focus on financial markets, microstructure

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Grossman and Stiglitz (AER, 1980)

- Joint normality of signals and payoffs
  + Exponential Utility =>
  - Quadratic objective function
  - Linear demand functions
  - Normally distributed prices and quantities
- Rational Expectations
  - Traders use correct models
- Three types of traders
  - Informed traders
  - Noise traders
  - Uninformed (but rational) traders

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Grossman and Stiglitz Model: Only Informed Traders

- Informed Traders
  - Continuum of identical traders
  - Risk Averse with exponential utility
  - Private signal = value + noise
  - Rational Expectations
  - Perfect Competitors
  - Condition on Price
- Could stop here (no other traders)
  - Price = weighted average of "signal" and "prior" mean
    - If $v \sim N(0,1/(1-T))$, $e \sim N(0,1/T)$, and $s = v+e$, then $E\{v|s\} = Ts$ and $var\{v|s\}=1$.
  - Constant discount for risk aversion if supply positive
  - Private information can be inferred from price
    - Information has no value.
  - No trade if traders begin with equal endowments
    - No trade with dynamic information process

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Milgrom and Stokey (JET, 1982)

- If …
  - Initial allocation of assets pareto optimal
  - Perfect competition
  - Rational expectations
- Then
  - No trade theorem
  - Prices fully revealing
- Is their setting of perfect competition necessary?
  - Can everybody expect to make a living playing poker (ma-jong)?
- Basic Idea: no noise trading (i.e. no losers) => no trade
- Information can be revealed dynamically

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Harris and Raviv (RFS, 1993)

- "Differences of Opinion Make a Horse Race"
  - Relax assumption that traders share common prior
  - Assume traders have different information
- Could get behavioral model by assuming two types of informed traders
  - Informed traders observe the same signal
  - Disagreement over correct parameters, e.g. signal-to-noise ratio in private information
  - Behavioral model based on type of disagreement
- Important Result: Trade occurs in equilibrium
  - More trade if more disagreement

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Grossman and Stiglitz Model: Add Noise Traders

- "Noise traders" : Who are they?
  - Hedging random endowment
  - Liquidity needs
  - "Bad" information (behavioral)
  - Enjoyment of gambling
- Optimization by noise traders not modeled.
- Noise not observed directly but maybe inferred from prices.
  - Problem that traders observe their own endowments
  - Diamond and Verrecchia (JFE, 1981) fix this.

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G & S Results:

- Informed traders profit at expense of noise traders.
- Prices incorporate random effect due to noise trading
- Excess volatility and mean reversion
- Calculations are essentially trivial.
- Meaningful concept of “market depth”

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Grossman and Stiglitz Model: Add Uninformed Traders

- Uninformed Traders = Market Makers?
  - Continuum of identical traders
  - Risk Averse with Exponential Utility
  - Continuum of Perfect Competitors
  - Condition on Price
- Problem is no longer trivial: How informative is price?
  - Need to condition on endogenous price
  - Price a mixture of signal (informed trade) and noise
  - Rational uninformed traders need a correct model of endogenous informativeness of price to equilibrium trading strategies.
    - Keynes got confused here.

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Market Microstructure Assumptions of G&S Model

- "Walrasian auctioneer" batches limit orders from informed traders, noise traders (market orders), and uninformed traders.
  - Equivalently, think of uninformed traders as market makers.
  - Anonymous trading: informed traders and noise traders look the same.
  - Equal access: All traders effectively have the same information about the trading process.
- Alternative to batching: orders arrive one-at-a-time
  - Glosten and Milgrom (JFE, 1985)
  - Incentives to mix order sizes to hide (Neuberger, 1970's)
  - Do arriving orders trade against market makers or limit order book?
    - Depends on how market is organized.

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G&S: More Results

- "Informativeness" of price and "bias" of price are different concepts
  - Informativeness (inverse): var{v|p}
  - Bias: E{v-p|p}
- Value of being an informed trader …
  - … does not depend on how many uninformed traders there are.
  - … does depend on number of noise traders and number of informed traders.
- With endogenous decision to become an informed trader, cost of private information determines informativeness of prices, not number of noise traders or market makers.
  - Impossibility of informationally efficient markets.
  - With risk neutral informed traders, equilibrium does not exist.

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Hellwig (JET, 1980)

- Generalize G&S to multiple informed traders.
- Math becomes more complicated.
- “Schizophrenia Problem”: Traders act like they have no effect on prices even though they do affect prices.

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Problem with Competitive Models

- Large traders affect prices and know they affect prices.
- G&S impossibility results if perfect competition assumed.
- Market liquidity does not directly affect traders’ incentives.
  - But “transactions costs” should affect traders’ behavior.

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Kyle (Econometrica, 1985): Single-Period Model

- Informed Trader
  - Monopolist, exercises monopoly power
  - Risk Neutral
  - Cannot condition on price
    - "Market order model": This model
    - "Limit Order Model": Alternative in Kyle (RES, 1989)
- Uninformed Traders = Market Makers
  - Perfect Competitors
  - Risk Neutral
  - Can condition on price
    - Take other side of market orders
- Noise Traders trade randomly as in G&S.
- Microstructure: Informed and noise trades batched, anonymously.

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An Equilibrium

- V = value ~ $N(0,\sigma^2_V)$
- U = noise ~ $N(0,\sigma^2_U)$
- Informed trader maximizes profits (quadratic problem):
  $dx = \beta*(V-P)*dt$
- Pricing Rule:
  $P = E\{V \mid \text{order flow} = dx + \text{noise}\}$
  $= \lambda * [\text{order flow}]$
- Equilibrium:
  $1/\lambda = [\text{market depth}] = \sigma^2_U / \sigma^2_V$
  $\beta = [\text{market depth}]/2$

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Intuition

- Monopolist reveals “half” of information to equate marginal cost of information (zero) with marginal revenue.
- Informed trader cares about market depth.
- Solution to “impossibility” result of G&S.

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Uniqueness

- Above equilibrium is the only one with linear pricing rule or linear trading strategy
  - In this equilibrium, non-linear pricing rules or trading strategies would not be more optimal
- My current research: The linear equilibrium is unique
  - i.e., there is no equilibrium with non-linear trading rule or pricing rule.
  - This is a very non-trivial result, known for my 1989 paper (e.g. Rochet and Vila) but not for 1985 paper.

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Are Assumptions Reasonable?

- Monopoly power:
  - Now seems obvious, but not in 1980.
  - Example: Salomon Brothers 2-year not squeeze.
- Risk neutrality:
  - Some hedge funds say they are “market constrained,” not “capital constrained.”
- Only market makers condition on price
  - Approximate mechanism to capture advantage of market makers with floor trading.
  - Time lags to cancel orders using telephones and hand signals

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Extensions

- Multiple informed traders:
  - Different information: Kyle (1981, 1984)
  - Same information:
- Market makers imperfect competitors
- Limit Order Model
- If informed trader committed to a strategy, it would be a different model.
  - In 1985 paper, no commitment to strategy.

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Concept of Market Liquidity: Traditional Empiricist

- Decompose bid-ask spread (from perspective of econometrician)
  - Order Processing Cost or Instantaneous Imperfect Competition: reverses immediately
  - Adverse Selection Component: permanent
  - Inventory Component: reverses slowly over time
    - Due to risk aversion of market makers
- G&S: both adverse selection and inventory component.
- Kyle (1985): only adverse selection component
  - Instantaneous reversal if market makers are imperfect competitors
  - Inventory component if market makers are risk averse

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Concept of Market Liquidity: Fischer Black (FAJ, 1971)

- Depth: impact for large orders
- Tightness: spread for small orders = Zero in G&S, Kyle
  - Commissions and fees would make this non-zero.
  - Black says should be almost zero
    - Electronic Platforms: Tightness almost infinite (sub-pennies), so focus should be on depth and resiliency
- Resiliency: time for uninformative order's effects to be reversed.
  - Resiliency is difficult to understand and hard to measure.
  - Need a dynamic model to understand resiliency.
- Is "slippage" the same as impact or a combination of impact and resiliency?

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Implications of Electronic Markets

- Easy to have electronic interface.
  - Market making with algorithmic trading, as predicted by Fischer Black
- Easy to conduct single-price auction.
- Easy to make credible data available.
- Easy to verify that rules are enforced
  - Maintain level playing field.
- Lower transactions costs leads to increased trading volume.
- Events on the ground are converging to theoretical models.

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Jack Treynor: Ahead of his time

- "The Only Game in Town" (FAJ, 1971)
  - Market makers spread represents liquidity trader losses, which are converted into informed trader profits.
- "What it Takes to Win the Trading Game" (FAJ, 1981???)
  - Behavioral explanation: Trading volume rises in bull market because traders incorrectly infer that making money implies they are smarter than average.

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Behavioral Biases and Transactions Costs

- Traders may under-estimate transactions costs.
  - Money managers
  - Customers of money managers
- Even if traders have as much private information as they think they have, "irrational" trading can occur if traders under-estimate transactions costs.
- Implications for mutual fund industry
  - A source of noise trading in stock market?

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Adding Behavioral Biases

- What if σ^2_V is the market makers' incorrect estimate of amount of private information?
  - In theoretical model, nothing changes, because σ^2_V is what the market makers think in equilibrium.
  - Empirically, if they are incorrect, they make or lose money.
- What if informed trader over-estimates market liquidity?
  - He trades too aggressively, and liquidity is less than formula says it is
  - Need to recalculate model.
- What if noise traders are irrational informed traders who think they have information but really do not have any?
  - Perhaps they observe noise but think it is information.
  - Markets are still "efficient" if market makers are risk neutral.
- Can be easy to calculate different models.

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"Market-Orders" versus "Limit Orders"

- 1985 paper – Market-order model
  - Captures idea that market makers have some flexibility that other traders do not have.
    - Traditional floor trading?
- 1989 paper – Limit-order model
  - Single-price auction
    - Can be viewed as an "auction theory" paper
  - Market makers = speculators with no private information, just risk-bearing capacity.
    - Electronic markets?
  - With many informed traders, obtain "monopolistic competition" in which each informed trader incorporates slightly less than half of signal into prices.
  - Traders restrict trading due to both risk aversion and exercise of monopoly power.

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Cash Settlement and Market Manipulation

- My current research in progress
- Cash settlement may create illusion of liquidity for naïve traders.
  - But liquidity cannot be created out of thin air.
- Traders have hedging motive to place large market on close orders when cash settled contracts expire.
  - To naïve regulator, these orders look like price manipulation, but they are really manifestations of perfect competition
  - The naïve trader who does not hedge cash settlement may be manipulating prices.

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Problems with One-Period Models

- One-period model of trade has three “prices”:
  P0 = prior mean
  P = equilibrium price at which trade occurs
  V = liquidation value
- Risk neutral market makers implies
  Var(V-P0) = var(P-P0) + var(V-P).
- How to define “Price Volatility” (with risk neutral market makers)
  - var(P-P0) = information revealed in price
  - or var(V-P) = information not revealed in price

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More Problems with One-Period Models

- Steady state effects: Error variance reduced as information process becomes better.
- Need concept of price autocorrelation (momentum)
- Changing state variables imply changing expected return
- Half-life of information (short-run versus long-run)
- Half-life of noise trading (short- and long-run)
- Dynamics of inventory models
- Dynamics of interaction between public and private information
- Dynamic competition between traders with different public information.
- Trade smoothing to reduce long-term market impact.

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Why Continuous-Time Model

- "No jumps" implies …
  - Mean-variance analysis (as in Merton)
  - Risk-free hedging (as in Black-Scholes)
  - Tractable PDE's instead of intractable difference equation systems (Kyle, 1985).
  - Incompatibility between CRRA and normal random variables (not log-normal)
    - Trader can bail out before he goes bankrupt.
  - "Walking the book" intuition is clear.
    - Alternative is smoothing trading over many auctions
  - Discipline in modeling time (ARCH, GARCH)

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Kyle (1985): Continuous Model

- Assumptions:
  - Noise traders' inventory follows brownian motion.
  - Informed trader observes signal at beginning of day; signal revealed publicly at end of day.
  - Auctions are held frequently, and in the limit continuously.
- Results:
  - Market depth = constant = $1/\lambda$ = same as one-period model
  - Informed Trader trades gradually
    - Information incorporated into prices at a constant rate
  - Prices follow brownian motion (from perspective of market maker).
    - From informed traders perspective, drift is exactly rate to make price fully revealing at end of day.
    - Volatility = constant = σ^2_V
  - Prices become fully revealing at end of day.
    - Informed trader makes twice as much money as in single period.
  - Continuous equilibrium is limit of model with discrete-time auctions.

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Kyle (1985): Continuous Intuition

- Informed trader trades gradually to “walk the book.”
- Prices fully revealing, so informed trader does not leave money on the table.
- Depth must be constant to prevent “manipulative” strategies
  - Push price to fully revealing now if depth about to fall.
  - Distort prices now and push back later is depth about to rise.
- Martingale implied by risk neutrality.
- Constant volatility since variance of noise trade is constant (and depth constant).

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Kyle (1985): Weaknesses of Model

- Informed trader faces no competition from public information
  - Is not in a hurry since prices reveal no information except his own trading.
  - All price fluctuations from “order flow effects” and not from “announcement effects.”
- Informed trader faces no competition from other informed traders with different information
- Noise traders should smooth their trading but do not do so.
  - Big savings from a small amount of smoothing.

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Fixing Weaknesses is Hard

- Townsend (JPE, 1983): “Forecasting the Forecasts of Others”
  - Infinite number of state variables needed in infinite horizon models where traders have different information
- Solution to forecasting-the-forecasts-of-others problem:
  - Hierarchical information: Foster and Viswanathan (JFQA, 1994).
  - Symmetric information: Foster and Viswanathan (JF, 1996).
  - Approximate Infinite Dimensions: Taub and Bernhardt (2003)

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Behavioral Biases: One Way to Be Rational, Many Ways … Irrational

- Buy on good news, sell on bad
  - Opposite of “buy rumor, sell news”
- Trend following
  - Buy after price rise
- Disposition effect
  - Hold on to losers, turn over winners
- Overconfidence
  - Think information more precise than it is.
  - Agreement to disagree about models
  - Kyle and Albert Wang (JF, 1997)
    - Overconfidence can lead to increased profits: acts as commitment device.
- Are market makers, informed traders, or noise traders irrational?

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Importance of Risk-Averse Market Makers

- If market makers are rational and risk neutral:
  - Market makers "undo" behavioral biases of others.
  - Returns are unpredictable.
  - There are no "anomalies."
- If market makers are rational and risk averse:
  - Market makers try to "undo" behavioral biases.
  - Does positively auto-correlated noise trading lead to momentum or to excess volatility and mean reversion?
    - Need a model to answer this question – not obvious
  - Excess volatility and mean reversion due to market maker risk aversion and demand for a risk-bearing service, not to irrationality per se.

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Summary

- My contribution
  - Imperfect competition solves problems with perfectly competitive models
  - Put a microstructure spin on rational expectations models
  - Exponential utility (or risk aversion) plus normal random variables = analytical tractability, even with imperfect competition

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