Market Microstructure in Continuous Time

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Problems with One-Period Models

- One-period model of trade has three "prices":
  P0 = prior mean
  P = equilibrium price at which trade occurs
  V = liquidation value
- Risk neutral market makers implies
  Var(V-P0) = var(P-P0) + var(V-P).
- How to define "Price Volatility" (with risk neutral market makers)
  - var(P-P0) = information revealed in price
  - or var(V-P) = information not revealed in price

More Problems with One-Period Models

- Steady state effects: Error variance reduced as information process becomes better.
- Need concept of price autocorrelation (momentum)
- Changing state variables imply changing expected return
- Half-life of information (short-run versus long-run)
- Half-life of noise trading (short- and long-run)
- Dynamics of inventory models
- Dynamics of interaction between public and private information
- Dynamic competition between traders with different public information.
- Trade smoothing to reduce long-term market impact.

Why Continuous-Time Model

- No jumps implies …
  - Mean-variance analysis (as in Merton)
  - Risk-free hedging (as in Black-Scholes)
  - Tractable PDE's instead of intractable difference equation systems (Kyle, 1985).
  - Discipline in modeling time (ARCH, GARCH)
  - Incompatibility between CRRA and normal random variables (not log-normal)
    - Trader can bail out before he goes bankrupt.

Kyle (1985): Continuous Model

- Assumptions:
  - Noise traders' inventory follows brownian motion.
  - Informed trader observes signal at beginning of day; signal revealed publicly at end of day.
- Results:
  - Market depth = constant = $1/\lambda$ = same as one-period model
  - Informed Trader trades gradually
    - Information incorporated into prices at a constant rate
  - Prices follow brownian motion (from perspective of market maker).
    - From informed traders perspective, drift is exactly rate to make price fully revealing at end of day.
  - Prices become fully revealing at end of day.
  - Volatility = constant = stdev(v)
  - Informed trader makes twice as much money.

Kyle (1985): Continuous Intuition

- Informed trader trades gradually to "walk the book."
- Prices fully revealing, so informed trader does not leave money on the table.
- Depth must be constant to prevent "manipulative" strategies
  - Push price to fully revealing now if depth about to fall.
  - Distort prices now and push back later is depth about to rise.
- Martingale implied by risk neutrality.
- Constant volatility since variance of noise trade is constant (and depth constant).

Kyle (1985): Weaknesses of Model

- Informed trader faces no competition from public information
  - Is not in a hurry since prices reveal no information except his own trading.
  - All price fluctuations from "order flow effects" and not from "announcement effects."
- Informed trader faces no competition from other informed traders with different information
- Noise traders should smooth their trading but do not do so.
  - Big savings from a small amount of smoothing.

Fixing Weaknesses is Hard

- Townsend (JPE, 1983): "Forecasting the Forecasts of Others"
  - Infinite number of state variables needed in infinite horizon models where traders have different information
- Solution to forecasting-the-forecasts-of-others problem:
  - Hierarchical information: Foster and Viswanathan (JFQA, 1994).
  - Symmetric information: Foster and Viswanathan (JF, 1996).
  - Approximate Infinite Dimensions: Taub and Bernhardt (2003)

Behavioral Biases: One Way to Be Rational, Many Ways … Irrational

- Buy on good news, sell on bad
  - Opposite of "buy rumor, sell news"
- Trend following
  - Buy after price rise
- Disposition effect
  - Hold on to losers, turn over winners
- Overconfidence
  - Think information more precise than it is.
  - Agreement to disagree about models
  - Kyle and Albert Wang (JF, 1997)
    - Overconfidence can lead to increased profits: acts as commitment device.
- Are market makers, informed traders, or noise traders irrational?

Importance of Risk-Averse Market Makers

- If market makers are rational and risk neutral:
  - Market makers "undo" behavioral biases of others.
  - Returns are unpredictable.
  - There are no "anomalies."
- If market makers are rational and risk averse:
  - Market makers try to "undo" behavioral biases.
  - Positively auto-correlated trading noise leads to excess volatility and mean reversion, not momentum!
  - Excess volatility and mean reversion due to market maker risk aversion, not to irrationality per se.

Literature

- Roll (1984): bid-ask bounce and negative first order autocorrelation
- Glosten and Milgrom (1985): no pooling of orders, efficient market
- Grossman and Miller(1988): risk-averse market makers, no private information
- Amihud and Mendelson (1986): bid-ask spread in discrete set-up

Microstructure Rules: More Complicated in Dynamic Setting

- Market Transparency = Level Playing Field?
  - Unequal access a big issue
- Time to place and cancel orders?
  - Limit order protection
- Who sees what in limit order book? Advantage to specialists or floor traders?
  - Transaction price history (with time lag for larger trades)
  - Current best bid and offer
  - Complete limit order book (anonymous)
  - Identities of traders. Ma, Lee, Hung (2004)
- Single-price auctions at open and close.

Tick Size in Dynamic Setting

- Practitioners and empiricists worry about shift to smaller tick size in US: 1/8's to 1/16's, to pennies.
- Large orders at discrete penny tick invite front running.
  - Solution is to submit scaled limit orders.
    - Many small limit orders at adjacent ticks.
- To protect against "winners curse" or "being picked off", offer little depth but revise orders frequently.
- Large trader effectively uses these principles in Kyle (1985)

Moral Hazard in Market Microstructure

- Customers demand fast service because they do not trust their brokers.
  - Fast trade reporting especially important
  - That is why people run around a traditional trading floor.
- Ticker was invented so that traders could monitor brokers.
- "Price fixings" (gold, LIBOR) allow some limit order protection when timing is important.

Measuring Depth

- Depth = Volume/Volatility
- Measurement of Volume
  - Number of Shares (per day)
  - Turnover as Percentage of Float (per day)
  - Dollar Volume (per day)
- Measurement of Volatility = Standard Deviation
  - Dollars per Share
  - Percentage of Value per Share
    - Black-Scholes as “implied” version

Measurement of Variance

- Empirical Variance = sum of squared price changes
- How to convert to expectation

Measurement of Volume

- Model calls for sum-of-squared-infinitesimal-quantities
  - This is a well-defined constant for brownian motion
  - Sum of absolute value of price changes would be infinite
- Suppose trade is auto-correlated
  - Need innovation of expected quantities bought or sold
- Should quantities be squared in Lee-and-Ready algorithm?
  - Problems if trades broken up into smaller quantities.

Other Aspects of Depth Measurement

- Does number of trades matter?
- Does trade size distribution matter?
- Need experiments to measure volatility accurately
  - Implementation shortfall requires careful calculation of “missed opportunity” when orders canceled.

Hellwig (JET, 1982): A Surprising Result

- Profitability of trading on private information does not necessarily go to zero in the limit as the difference between the "fully revealing" price and the actual price goes to zero.
- Hellwig's paper is not very intuitive
- Intuition: Depth is what matters.
  - Each period dt, noise trading creates error variance of order dt
  - Informed trader drives price almost all the way fully revealing value over a short period of time, walking down demand curve, trading quantity proportional to depth and of order sqrt(dt).
  - Resulting profit is of order dt for each period of length dt, so profits integrate to an amount proportional to depth.
- Ming Guo (2004) and Kyle and Guo (2004) have different model making similar point.

Cash Settlement and Market Manipulation

- Cash settlement may create illusion of liquidity for naïve traders.
  - But liquidity cannot be created out of thin air.
- Traders have hedging motive to place large market on close orders when cash settled contracts expire.
  - To naïve regulator, these orders look like price manipulation, but they are really manifestations of perfect competition
  - The naïve trader who does not hedge cash settlement may be manipulating prices.

How to Model Dynamic Information

- Public Information: Dividends or Earnings or Cash Flows
- Private Information: Signal about future dividends
- Public information makes private information decay.
- Different sources of private information might make each other decay, if the signals lead to trade.

Example of Dynamic Information: Campbell & Kyle (RES, 1993)

- Information "signal" I(t)
  - mean-reverting AR-1 = O-U process
  - dI = -a*I*dt + b*dWI
- Dividend "drift" is I(t)
  - dD = k*I*dt + c*dWD
- Innovations dWI and dWD correlated so that univariate process for dD is a brownian motion without drift.
  - i.e., E{I(t) | history of D(s)} = 0.

Campbell and Kyle: Assumptions

- No market makers
  - Jiang Wang (RES, 1993) adds market makers.
- Informed traders = Value-Based Investors
  - Risk aversion: continuous exponential utility
- Noise traders' inventory follows O-U process:
  - dU = -f*U*dt + h*dWU
  - "over-reaction if dU correlated with dP or dD

Campbell and Kyle: Results

- Prices are predictable
  - Sharpe-ratio is linear in noise traders' inventory.
- Excess volatility and mean reversion.
  - Hard to obtain momentum in this type of model.
- Informed traders have hedging demand.
- Econometrician faces kalman filtering problem similar to what a market maker would face.
  - Can estimate model on price and dividend data.
  - Noise trade identified by sharpe ratio.

Even Simpler Behavioral Model

- Only type of trader is “informed” trader = representative investor
  - No noise trade or market makers.
- Similar information process.
- “Informed” trader uses incorrect parameters.
  - Overconfidence: Assumed k too large
    - Excess volatility and mean reversion
  - Under-confidence: Assumed k too small
    - Too little variance and momentum

Behavioral Approach

- Tao Lin (2003) and Kyle and Lin (2003):
  - No noise trading
  - Symmetrically informed traders
  - Traders are overconfident, have differences in beliefs, agreement to disagree about differences.
    - Each trader thinks his model is better than the other traders.
  - Results:
    - Trading volume proportional to degree of overconfidence.
    - Prices aggregate information.
  - Detemple and Murthy (JET, 1994)
    - Same idea in log-utility model: information shows up in interest rates.

Overconfidence and Short-Sale Constraints

- Detemple and Murthy (RFS, 1997), Scheinkman and Xiong (JPE, 2004)
  - Price bubbles since traders believe they can sell asset to a less informed trader.
  - Models allow large degree of price inflation above fundamental value.

Frontiers of Research

- Why is there so much trading volume?
  - Behavioral Biases.
- National Market System:
  - Interfaces between exchanges
  - Slow mechanisms versus fast mechanisms
    - Using slowness to protect a market against attack.
- Limits of Arbitrage: What forces keep prices efficient
- Causes and effects of order smoothing
- Equilibrium with proportional transactions costs

Where We Are Today

- Behavioral Assumptions
- Numerical Methods

For Everything I left Out…

- Read Larry Harris's book!

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